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Sample Solution

Question	1	2	3	4	5	6	7	8	9	10	11	12	Total
Max	8	10	7	7	7	10	12	7	8	10	7	7	100
Earned													

Question 1: [8 Points] [CLO 1] Propositional Logic

Construct a truth table for the compound propositions: $(p \rightarrow q) \vee (\neg p \rightarrow q)$

p	q	$p \rightarrow q$	$\neg p$	$\neg p \rightarrow q$	$(p \rightarrow q) \vee (\neg p \rightarrow q)$
F	F	T	T	F	T
F	T	T	T	T	T
T	F	F	F	T	T
T	T	T	F	T	T

Question 2: [10 Points] [CLO 1] Applications of Propositional Logic

Show solution steps to find whether these system specifications are consistent?

- Whenever the system software is being upgraded, users cannot access the file system.
- If users can access the file system, then they can save new files.
- If users cannot save new files, then the system software is not being upgraded.

p = "The system software is being upgraded"
q = "Users can access the file system"
r = "Users can save new files"

$p \longrightarrow \neg q$
$q \longrightarrow r$
$\neg r \longrightarrow \neg p$

An assignment that makes the clauses all true
p = FALSE, q = TRUE, and r = TRUE.
So the system specification is consistent.
The truth table is only for illustration.

p	q	r	(p→¬q) ∧ (q→r) ∧ (¬r→¬p)
T	T	T	F
T	T	F	F
T	F	T	T
T	F	F	F
F	T	T	T
F	T	F	F
F	F	T	T
F	F	F	T

Question 3: [7 points] [CLO 1] Propositional Equivalences
Show that $(p \to q) \to (r \to s)$ and $(p \to r) \to (q \to s)$ are not logically equivalent.

Let r be true and p, q, and s be false, then $(p \to q) \to (r \to s)$ will be false, but $(p \to r) \to (q \to s)$ will be true. The truth table is only for illustration.

p	q	r	s	(p → q) → (r → s)	(p → r) → (q → s)
F	F	F	F	T	T
F	F	F	T	T	T
F	F	T	F	**F**	**T**
F	F	T	T	T	T
F	T	F	F	**T**	**F**
F	T	F	T	T	T
F	T	T	F	F	F
F	T	T	T	T	T
T	F	F	F	T	T
T	F	F	T	T	T
T	F	T	F	T	T
T	F	T	T	T	T
T	T	F	F	T	T
T	T	F	T	T	T
T	T	T	F	F	F
T	T	T	T	T	T

Question 4: [7 points] [CLO 1] Predicates and Quantifies
Translate the following statement into logical expressions using predicates, quantifiers, and logical connectives.

Everything is in the correct place and in excellent condition.

$\forall x\ (C(x) \wedge E(x))$.

Question 5: [7 points] [CLO 1] Nested Quantifiers
Rewrite the following statement so that negations appear only within predicates (that is, so that no negation is outside a quantifier or an expression involving logical connectives).

$\neg \exists y(Q(y) \wedge \forall x \neg R(x, y))$

$\forall y(\neg Q(y) \vee \exists x\ R(x, y))$

Question 6: [10 points] [CLO 2] Rules of Inference

Identify the error or errors in this argument that supposedly shows that if $\forall x(P(x) \vee Q(x))$ is true then $\forall x P(x) \vee \forall x Q(x)$ is true.

1. $\forall x(P(x) \vee Q(x))$ Premise
2. $P(c) \vee Q(c)$ Universal instantiation from (1)
3. $P(c)$ Simplification from (2)
4. $\forall x P(x)$ Universal generalization from (3)
5. $Q(c)$ Simplification from (2)
6. $\forall x Q(x)$ Universal generalization from (5)
7. $\forall x(P(x) \vee \forall x Q(x))$ Conjunction from (4) and (6)

There are four errors. First, simplification is the rule $(p \wedge q) \rightarrow p$; the rule does not apply to $\vee$. This causes an error on lines 3 and 5. The third and the fourth error are on the final line. The conjunction rule works with $\wedge$ operator not $\vee$ operator and it is misapplied — look closely at the placement of the parentheses.

Question 7: [12 points] [CLO 2] Introduction to Proofs

Prove that $m^2 = n^2$ if and only if $m = n$ or $m = -n$.

- We need to show that: $(m^2 = n^2) \leftrightarrow [(m = n) \vee (m = -n)]$

- Need to prove two parts:
  - Part 1: $[(m = n) \vee (m = -n)] \rightarrow (m^2 = n^2)$
    - Case 1: $(m = n) \rightarrow (m^2 = n^2)$
      - $(m)^2 = n^2$, and $(n)^2 = n^2$, so this case is proven
    - Case 2: $(m = -n) \rightarrow (m^2 = n^2)$
      - $(m)^2 = m^2$, and $(-n)^2 = n^2$, so this case is proven
  - Part 2: $(m^2 = n^2) \rightarrow [(m = n) \vee (m = -n)]$
    - Subtract n^2 from both sides we get $m^2 - n^2 = 0$
    - Factor to get $(m + n)(m - n) = 0$
    - Since that equals zero, one of the factors must be zero
    - Thus, either $m + n = 0$ (which means $m = n$) or $m - n = 0$ (which means $m = -n$)

Question 8: [7 Points] [CLO #1] Set Operations

Suppose that the universal set is $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$.
Express the set $\{1, 3, 6, 10\}$ with bit strings where the ith bit in the string is 1 if i is in the set and 0 otherwise.

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Question 9: [8 Points] [CLO #2] Functions

Determine whether the function $f(x) = 2x + 1$ is a bijection from **R** to **R**.

YES as it is one to one and Onto.

Question 10: [10 Points] [CLO #3] Sequences

For the following list of integers, provide a simple formula or rule that generates the terms of an integer sequence that begins with the given list. Assuming that your formula or rule is correct, determine the next four terms of the sequence.

1, 2, 2, 2, 3, 3, 3, 3, 3, 5, 5, 5, 5, 5, 5, 5, . . .

The pattern is made up of Fibonacci numbers starting with one 1, the three 2's, then five 3's, and so on. The number of copies increase by two each time. The next three terms are 8, 8, 8.

Question 11: [7 Points] [CLO #3] Summations

Compute the following double sums.

$$\sum_{i=0}^{2}\sum_{j=0}^{3} i^2 j^3$$

$$\sum_{i=0}^{2}\sum_{j=0}^{3} i^2 j^3 = \sum_{i=0}^{2} i^2 \sum_{j=0}^{3} j^3 = \sum_{i=0}^{2} i^2(0^3 + 1^3 + 2^3 + 3^3) = \sum_{i=0}^{2} i^2(1 + 8 + 27)$$

$$= \sum_{i=0}^{2} i^2(36) = 36\sum_{i=0}^{2} i^2 = 36\sum_{i=0}^{2}(0^2 + 1^2 + 2^2) = 36(5) = 180$$

Question 12: [7 Points] [CLO #3] Cardinality of Sets

Determine whether the set of the integers with absolute value less than 1,000,000 is finite, countably infinite, or uncountable. If the set is countably infinite, exhibit a one-to-one correspondence between the set of positive integers and that set.

Finite, of cardinality $2 * (1,000,000 - 1) + 1$.